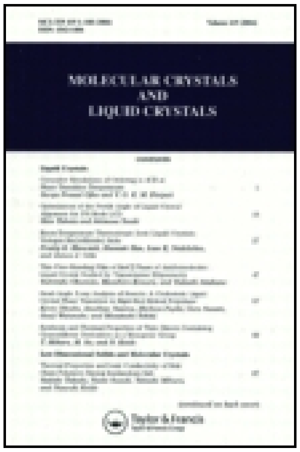

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

Surface Guided Electromagnetic Waves of Higher Diffraction Orders in Cholesterics

V. A. Belyakov [a] & G. I. Shilina [a]

[a] All-Union Surface and Vacuum Research Centre, Andreevskaya nab. 2, 117334, Moscow, USSR

Version of record first published: 04 Oct 2006.

To cite this article: V. A. Belyakov & G. I. Shilina (1992): Surface Guided Electromagnetic Waves of Higher Diffraction Orders in Cholesterics, Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals, 223:1, 55-67

To link to this article: http://dx.doi.org/10.1080/15421409208048240

Mol. Cryst. Liq. Cryst. 1992, Vol. 223, pp. 55–67
Reprints available directly from the publisher
Photocopying permitted by license only

Printed in the United States of America

SURFACE GUIDED ELECTROMAGNETIC WAVES OF HIGHER DIFFRACTION ORDERS IN CHOLESTERICS

V.A.BELYAKOV and G.I.SHILINA
All-Union Surface and Vacuum Research Centre, Andreevskaya nab.2, 117334 Moscow, USSR

(Received October 10, 1991)

Abstract Optical surface guided electromagnetic waves (SGEW) of the second order diffraction in cholesterics are examined in the framework of dynamical diffraction theory. The dispersion equations are obtained and analyzed. Allowed SGEW frequency band and directions of propagation (relative to the orientation of director at the cholesteric surface) are found. The dependence of SGEW penetration in cholesteric on the frequency and propagation direction is investigated. It is shown that two of the four different SGEW modes of the second diffraction order have unusual polarization properties changing with the SGEW frequency.

INTRODUCTION

Surface guided electromagnetic waves (SGEW) at an interface of homogeneous and periodic media are due to total internal reflection at the boundary with the homogeneous media and diffraction reflection from the bulk of periodic media. SGEW at cholesteric surfaces for the first order of diffraction were investigated by several authors[1-3]. In the cited papers restrictions on the SGEW propagation directions (relative to the orientation of director at the cholesteric surface) and its unusual polarization (elliptical in the general case) were found. However, the mentioned results were obtained by the numerical methods because of mathematical complexity of the problem.

In the present paper SGEW of the second order diffraction in cholesterics are examined and an approximate solution valid in the problem is found in an analytical form. This solution simplifies general analysis of the problem as well as analysis of specific cases. The simplicity of the solution for SGEW in second (and higher) order diffraction is explained by decoupling of linear polarizations (of corresponding equations)in the eigenmodes for the higher orders diffraction optical problem for cholesterics[4] .

BASIC EQUATIONS

We examine SGEW at an interface of a homogeneous media and a planar cholesteric texture due to the second order Bragg diffraction. We use a coordinate system in which a semi-infinite isotropic homogeneous medium with dielectric constant ε_1 lies in the region for z>0, the semi-infinite cholesteric medium in the region for z<0, and their interface coincides with the x-y plane. The cholesteric dielectric tensor is given by the formula:

$$\hat{\varepsilon} = \begin{bmatrix} \bar{\varepsilon} + \bar{\varepsilon}\delta\cos(\tau z-2\phi) & \bar{\varepsilon}\delta\sin(\tau z-2\phi) & 0 \\ \bar{\varepsilon}\delta\sin(\tau z-2\phi) & \bar{\varepsilon} - \bar{\varepsilon}\delta\cos(\tau z-2\phi) & 0 \\ 0 & 0 & \varepsilon_\perp \end{bmatrix}, \quad (1)$$

where $$\bar{\varepsilon} = \frac{\varepsilon_\parallel + \varepsilon_\perp}{2}; \quad \delta = \frac{\varepsilon_\parallel - \varepsilon_\perp}{\varepsilon_\parallel + \varepsilon_\perp}$$

$\varepsilon_\parallel$, $\varepsilon_\perp$ are the principal values of the dielectric tensor, τ is the reciprocal lattice vector of the cholesteric structure, which is connected with the pitch p by the relation $\tau = 4\pi/p$, ϕ is the angle between x-axis and the director at the cholesteric surface, δ is the dielectric anysotropy.

The approximate SGEW field valid in the present problemwill be in the form (the factor exp(-iωt) is omitted below):

$$E = E_1 e^{iqx - \gamma_1 z} , \quad z>0, \quad \gamma_1 = \sqrt{q^2 - \bar{\varepsilon}_1 \frac{\omega^2}{c^2}} \qquad (2)$$

$$E = \sum_{j=1}^{2} C_j (E_{oj}\, e^{i(t_j - \tau)z} + E_{2j}\, e^{i(t_j + \tau)z})\, e^{iqx}, \quad z<0. \qquad (3)$$

where in (3) the summation is carried out over the optical eigenmodes in the neighbourhood of second order diffraction condition [4], C_j are the coefficients to be determined together with the SGEW polarization and wave vectors q and the diffraction corrections to their z-component t_j.

Note that there is a threshold frequency ω_{min} for SGEW, which for the second order diffraction is determined by the formulae $\omega_{min} = c\tau/(\bar{\varepsilon}-\varepsilon_1)^{1/2}$. SGEW of frequencies below ω_{min} does not exist.

It is known [4] that the optical eigenmodes for second order diffraction are practically linearly polarized and E_{oj}, E_{2j} almost coincide with linear σ and π polarization (σ and π correspond to linear polarizations in the plane of diffraction scattering and perpendicular to this plane, respectively).

Therefore four possibilities exist for the eigenmode polarization structure :

1. E_{oj} and E_{2j} both are σ- polarized
2. E_{oj} and E_{2j} both are π- polarized
3. E_{oj} and E_{2j} are σ- and π- polarized, respectively
4. E_{oj} and E_{2j} are π- and σ- polarized, respectively

σ- AND π - POLARIZED SGEW

The simplest solutions for SGEW are for the cases 1 and 2, in which each of the eigenmodes is excited in the cholesteric. In each of these cases, SGEW are either σ- or

π- polarized.

Examine the case 1 of σ- polarized SGEW. In this case the expression (3) reduces to ($t_j = i\gamma$ are purely imaginary):

$$E_\sigma = C\left(e^{i\frac{2\phi+\beta}{2}} e^{-(\gamma+i\tau)z} + e^{-i\frac{2\phi+\beta}{2}} e^{-(\gamma-i\tau)z}\right) e^{iqx} \qquad (4)$$

where

$$q = q_B + \Delta q\ ; \quad q_B^2 = \varkappa_\sigma^2 - \tau^2 \qquad (5)$$

$$\varkappa_\sigma^2 = \bar{\varepsilon}\,\frac{\omega^2}{c^2}\left(1 + \frac{\delta}{2}\cos^2\psi_B\right) \qquad (6)$$

The parameter ψ_B is determined by the relation

$$\sin^2\psi_B = \frac{\tau^2}{\varkappa_\sigma^2}\ , \qquad (7)$$

Δq and γ are connected by the relations

$$2q_B\Delta q = \frac{\delta^2}{4}\,\varkappa_\sigma^2\,\mathrm{ctg}^2\psi_B\cos\beta$$
$$2\tau\gamma = \frac{\delta^2}{4}\,\varkappa_\sigma^2\,\mathrm{ctg}^2\psi_B\sin\beta\ , \quad -\pi < \beta < 0 \qquad (8)$$

where $\cos\beta$ determines the deviation from the Bragg condition in two-wave approximation[4] and is analogous to the parameter α in the cited work.

The dispersion equation for SGEW is obtained from the continuity conditions for tangential components of the electric and magnetic fields at the cholesteric boundary (z=0). In zero approximation at δ^2 the dispersion equation for σ- polarized SGEW takes the form:

$$\mathrm{tg}\,\frac{2\phi+\beta}{2} = -\frac{\gamma_1}{\tau} \qquad (9)$$

where

$$\gamma_1 = \sqrt{(\bar{\varepsilon} - \varepsilon_1)\frac{\omega^2}{C^2} - \tau^2 + \frac{\delta}{2}\,\bar{\varepsilon}\frac{\omega^2}{C^2}\cos^2\psi_B} \qquad (10)$$

In the analogous way, one obtains the dispersion equation for π- polarized SGEW

$$\mathrm{tg}\,\frac{2\phi+\beta}{2} = -\frac{\bar{\varepsilon}\,\gamma_1}{\varepsilon_1\tau}\left(1 - \frac{\delta}{2}\cos^2\psi_B\right) \qquad (11)$$

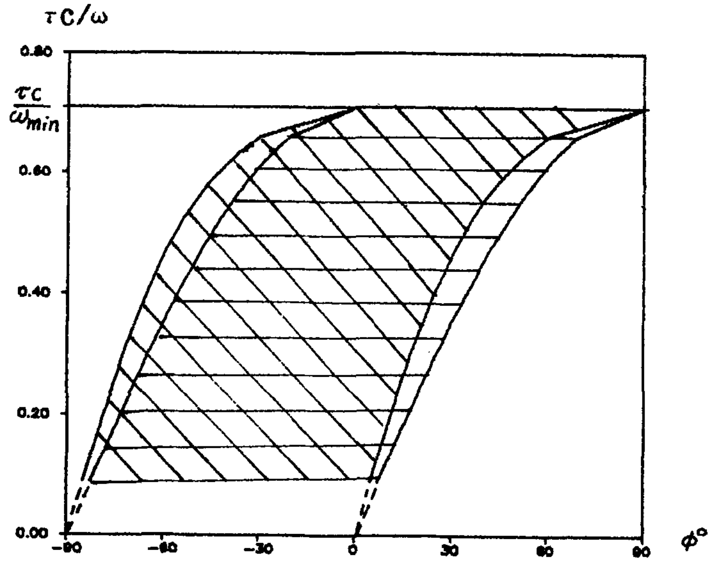

FIGURE 1 Calculated allowed regions of SGEW
- σ- polarization, - π- polarization
$\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$

Graphs of the allowed regions in coordinates $\lambda - \phi$ (λ being the wave length and ϕ the angle between the propagation direction and the director at the cholesteric surface) for σ- and π- polarized SGEW are presented in Fig.1. The σ- and π- polarized SGEW of second order diffraction are described practically in the same way as SGEW in scalar periodic media[5-7].

Note that the role of the phase of dielectric constant modulation in the case of cholesteric plays the angle between the propagation direction and director at the cholesteric surface (see Fig.2). For any allowed wavelength, the σ- and π- polarized SGEW in cholesteric propagate in an angular sector equal to $\pi/2$ the position of which relative to the director orientation at the cholesteric surface depends on the frequency of SGEW.

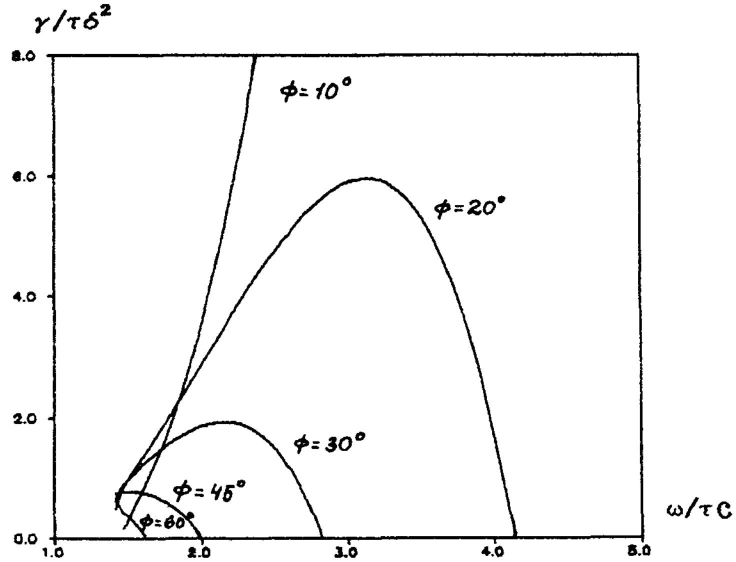

FIGURE 2 Calculated frequency dependence of damping of the σ- polarized SGEW field for different propagation directions.
$\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$

SGEW OF MIXED POLARIZATIONS

If the eigenmodes corresponding to either case3 or 4 are excited in cholesterics, the solution for SGEW is more complex. In each of these cases, two eigenmodes are included in the sum given by Eq.(3). Now the parameters t_j and Δq are connected by the following relations:

$$t_1 = -t_2^* = -\frac{q_B^2}{2\tau}\left(\frac{\delta}{2} - i\frac{\delta^2}{4}\,\frac{\sin\beta}{\sin\psi_B}\right)$$

$$\Delta q = -\frac{\delta^2}{8}q_B\left(\frac{q_B^2}{4\tau^2} + \frac{\delta^2}{4}\,\frac{\cos\beta}{\sin\psi_B}\right) \qquad (12)$$

In zeroth order approximation at δ^2, in the same way as above, one obtains the following dispersion equation:

$$tg(2\phi+\beta) = \frac{\gamma_1\left[2\tau^2(\bar{\varepsilon} + \varepsilon_1)\frac{\omega^2}{c^2} + \delta q_B^4\right]}{\tau\left[2(\gamma_1^2\bar{\varepsilon} - \varepsilon_1\tau^2)\frac{\omega^2}{c^2} - \delta q_B^4\right]} \qquad (13)$$

The graph of the allowed region for the SGEW of mixed polarizations following from Eq.(13) is given in Fig.3. In this case there are two SGEW modes of different polarizations. Each of these modes for any allowed frequency propagates in an angular sector equal to $\pi/2$. The positions of these sectors relative to the director orientation at

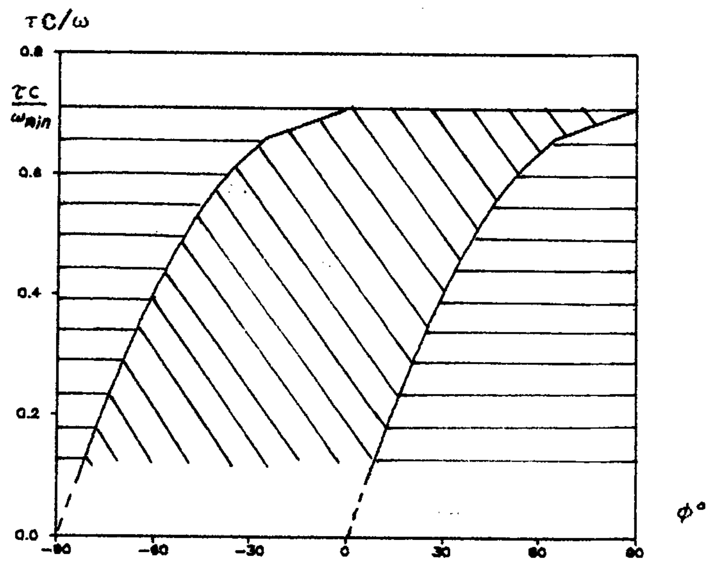

FIGURE 3 Calculated allowed regions of mixed polarization SGEWs. $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$

the cholesteric surface change with the frequency but the corresponding sectors are complementary toone another so that their angular sum is equal to π for all frequencies, perhaps with the excepton a close neighbourhood of the minimal frequency ω_{min}. It means that, in the sum for both modes, all the directions relative tothe director are allowed for SGEW propagation.

The mixed polarization SGEW modes are linearly polarized but their polarizations are different from σ- and π- polarizations and change with the SGEW frequency (linear is the polarization of the field in homogeneous medum). The angle ξ which makes the plane of polarization

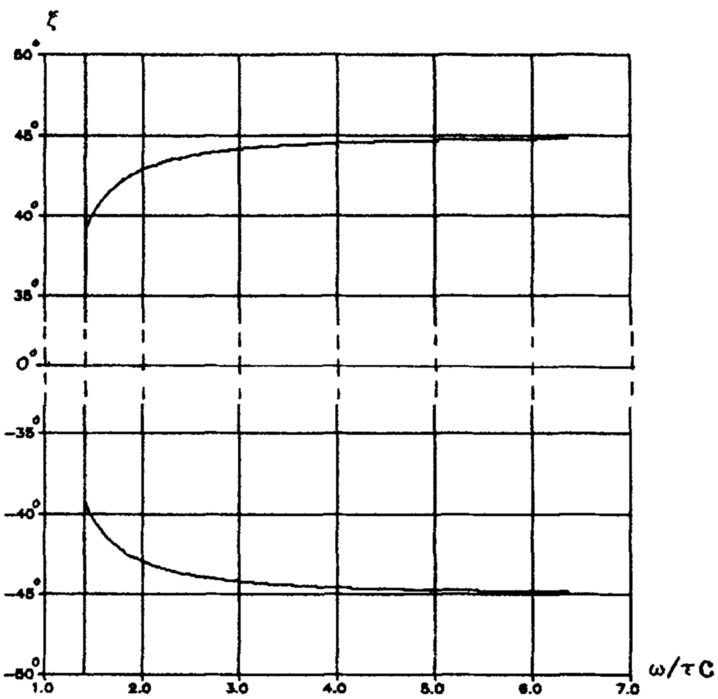

FIGURE 4 Calculated frequency dependence of the angle ξ between the plane of polarization of SGEW modes and the normal to cholesteric surface for the mixed polarization SGEW. $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$

with the normal to the cholesteric surface is determined by:

$$\mathrm{ctg}\,\xi = \frac{q_B}{\gamma_1}\sin\psi_B[\sin(2\phi+\beta) + \frac{\gamma_1}{\tau}\cos(2\phi+\beta)] \qquad (14)$$

The dependence of the angle ξ on frequency for the both modes is shown in Fig. 4

Note that the SGEW fields are strongly dependent on the angle between the propagation direction and the director at the cholesteric surface (z=0). The dependence of Δq on ϕ is shown in Fig.5. The dependence of damping (γ) of the SGEW field on ϕ is chown in Fig.6. The values of ϕ for which γ reduces to zero correspond to points on the boundary of the allowed region of SGEW. The frequency dependence of γ varies also with the SGEW propagation direction (see Fig. 7). The dependence of Δq on frequency for the mixed polarization SGEW modes are shown in Fig.8.

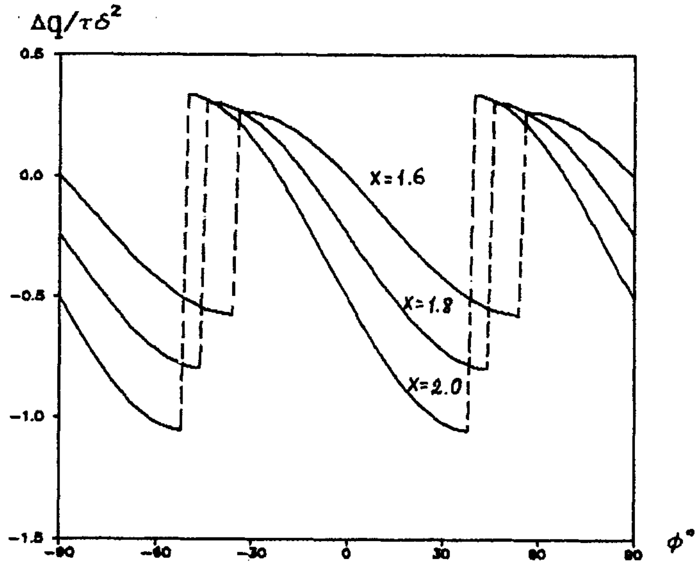

FIGURE 5 Calculated dependence of Δq (correction to the SGEW wave vector) for the mixed polarization SGEWs on the direction of SGEW propagation for different frequencies. $\bar{\varepsilon}$ = 1.5 , ε_1 = 1.0 , δ = 0.05, $x = \omega/\tau c$

CONCLUSION

The analytical investigations of SGEW of second order diffraction in cholesterics carried out above reveal some peculiarities of these waves. First of all, they relate to their polarization properties. There are four different SGEW modes which are linearly polarized and reasonably accurate expressions of the SGEW fields for the present calculation.. Two of them are σ– or π– polarized SGEW for which allowed propagation directions relative to the director at the cholesteric surface makes an angular sector equal to $\pi/2$, thus forbidden for propagation of these SGEW modes directions exist. The other two modes are linearly polarized at an angle to the normal to the cholesteric surface. The allowed propagation sectors for each of them are also $\pi/2$ but are complementary toone another, so

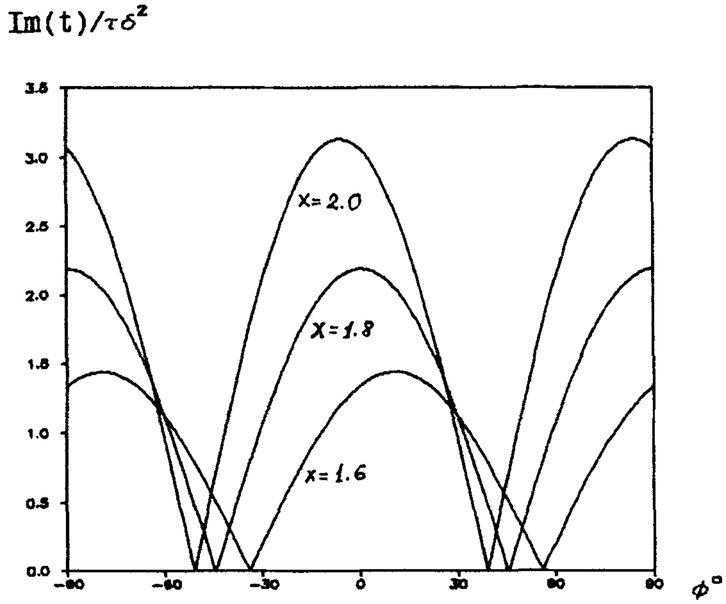

FIGURE 6 Calculated damping of the mixed polarization SGEW field in the depth of cholesteric versus the SGEW propagation direction for different frequencies. $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$, $x=\omega/\tau c$

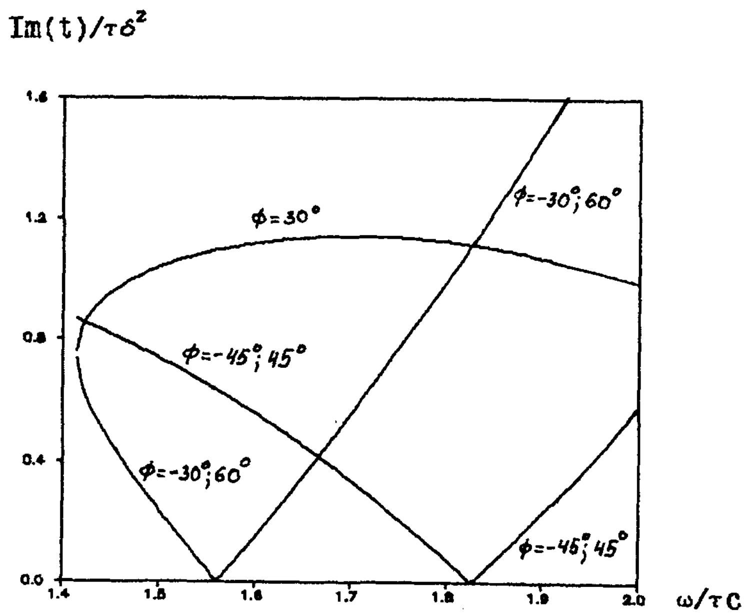

FIGURE 7 Calculated frequency dependence of the mixed polarization SGEW field damping in the cholesteric depth for different propagation directions. $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$

thatall directions of propagation relative to the director at the cholesteric surface are covered by these two modes. Note that the orientations of the planes of polarizations of these two SGEW modes are changing with the frequency. The relative changes of the wave vector q for all these four modes are of the order of δ. All the parameters of SGEW of the second order diffraction except forpolarizations are strongly dependent on the SGEW propagation direction.

The rich variety of properties of the SGEW of the second diffraction order in cholesteric together with the possibility of their analytical description make them an intriguing object for experimental investigation.

The properties of the mixed polarization SGEW modes deserve special attention. Since it is possible to change

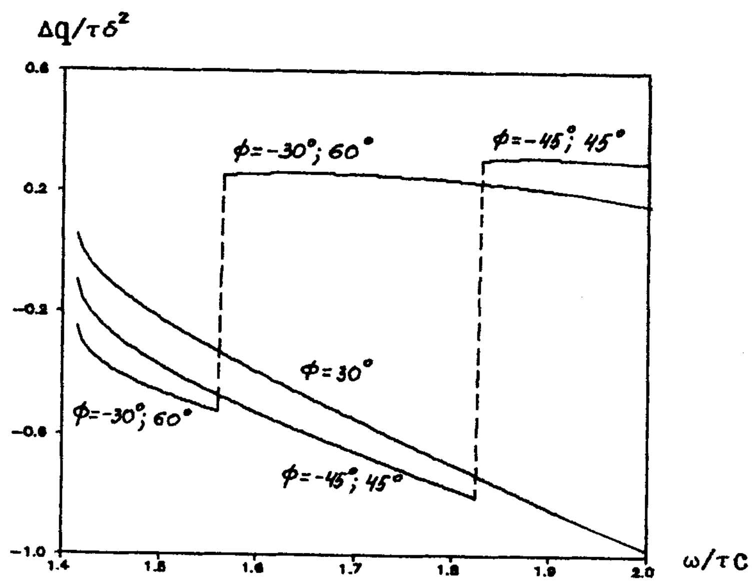

FIGURE 8 Calculated frequency dependence of Δq (correction to the SGEW wave vector) for the mixed polarization SGEWs for different propagation directions $\bar{\varepsilon}$ = 1.5, ε_1 = 1.0, δ = 0.05

the cholesteric structure easily by a weak external force, the SGEW in cholesterics appears to be promising for the applications, but here again the last words should come from the experimentalists.

REFERENCES

1. S.V.Shiyanovskii, Zurnal Tekhnicheskoi Fiziki , 57, 1448 (1987). S.V.Shiyanovskii,MCLC,179, 133 (1990)
2. V.V.Popov , Kristallografiya , 32, 984 (1987) V.V.Popov , Zurnal Tekhnicheskoi Fiziki , 5, 2396 (1986)
3. V.A.Belyakov , V.P.Orlov , Poverkhnost, 1,13 (1990) V.A.Belyakov , V.P.Orlov , MCLC Lett.,8, 1 (1991)
4. V.A.Belyakov , Diffraction Optics of Complex Structured Media,(Nauka, Moscow, 1989(in Russ, to be translated into English by Springer Verlag)),

V.A.Belyakov, V.E. Dmitrienko, Optics of Chiral Liquid Cristals (Harwood Academic Publishers, 1989 (Sov.Phys.Rev. ed. by.I.M.Khalatnikov))

5. P.Yeh, A.Yariv , A.Y.Cho , Appl.Phys.Lett., 32, 104 (1978)
6. A.V.Vinogradov, I.V.Kozhevnikov, Pisma Zurnal Exp. Theor.Fiz., 40, 405 (1984)
7. V.A.Belyakov , V.P.Orlov, G.I.Shilina, Zurnal Exp Theor Fiz.,101 , 6 (1992) (To be published)